MedCORDEX

Workshop on PDE's
Modelling & Theory
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The Mediterranean Sea level variability from an ensemble of numerical model simulations and analytical expressions

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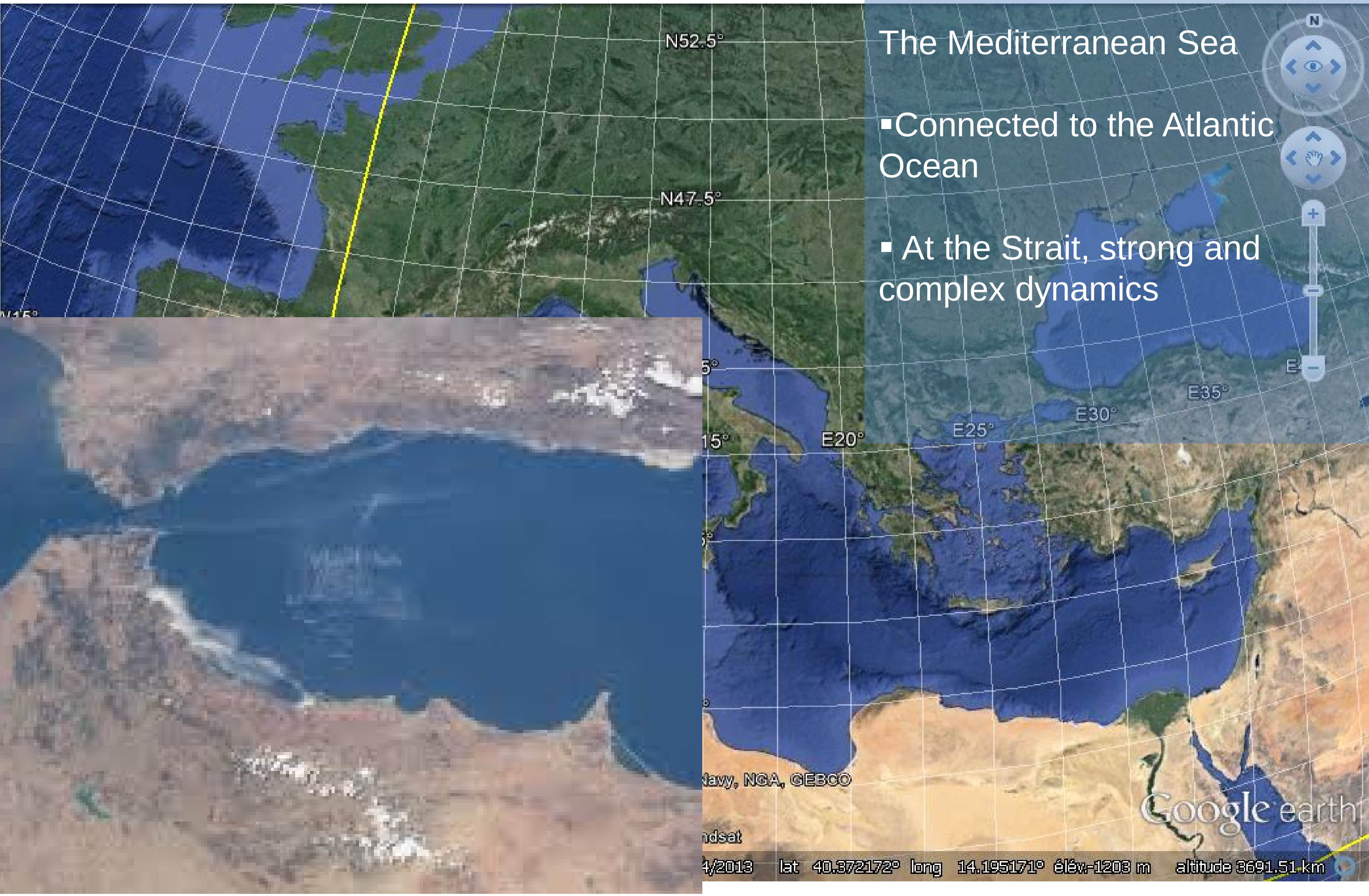
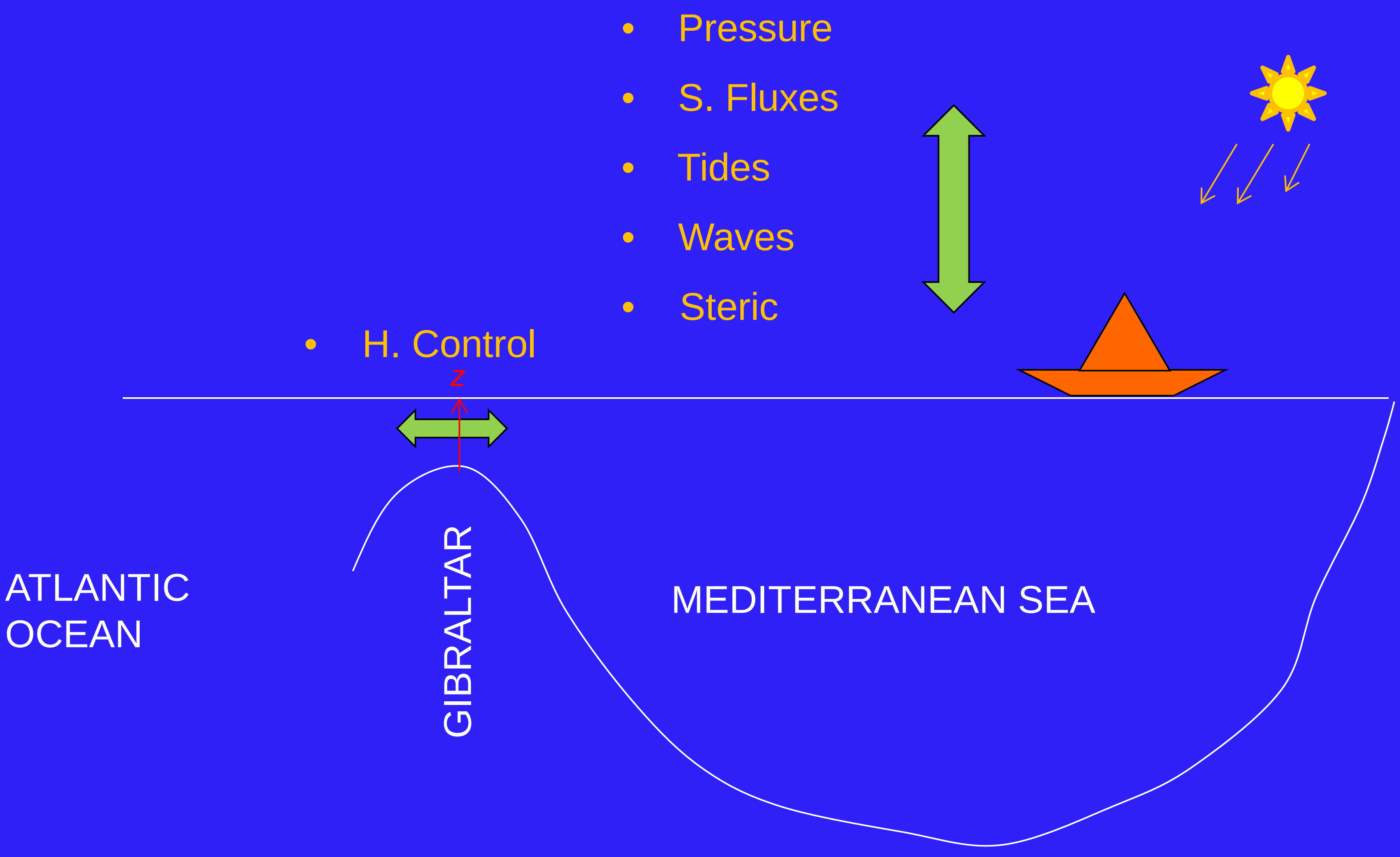


The Mediterranean Sea

- Connected to the Atlantic Ocean
- At the Strait, strong and complex dynamics



Pressure
S. Fluxes
Tides
Waves
Steric
H. Control
z
ATLANTIC OCEAN
GIBRALTAR
MEDITERRANEAN SEA

The Velocity equation (Harzallah,2009)

The equation of motion: $$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \lambda u.$$ Candela et al (1989)

u is the velocity at the Strait
p is pressure
λ is a friction coefficient

Transformed to: $$\frac{\partial u}{\partial t} = -g\frac{\eta}{L} - \frac{P_a}{\rho_0 L} + g\frac{\Delta\rho}{\rho_0}\frac{z}{L} - \lambda u,$$ Harzallah (2009)

η is the Med. Sea level
Pa is the atmospheric pressure
Δρ is the density difference along the Strait with length *L*

The Velocity equation (Harzallah,2009)

We consider: $$-\lambda u = -\lambda_0 u + \lambda_0 \alpha \frac{z}{H_G} u.$$

with: $$\langle \lambda \rangle = \lambda(1+\alpha/2)$$

λ_0 is at z=0

< λ > Is the vertical average of λ with a slope α

We obtain: $$\frac{\partial u}{\partial t} = -\frac{g\eta}{L} - \frac{P_a}{\rho_0 L} + \frac{g}{L}\frac{\Delta\rho}{\rho_0} z - \lambda_0 u + \frac{\alpha\lambda_0 Q}{H_G A_G} z \qquad (1)$$

Q is the transport

HG is the Strait depth

AG is the Strait width

The Velocity equation (Harzallah,2009)

Integrated over the Strait: $\int_{AG} (1) ds$

$$\frac{1}{A_G}\frac{\partial Q}{\partial t} = -\frac{g\eta}{L} - \frac{P_a}{\rho_0 L} - \frac{gH_G}{2L}\frac{\Delta\rho}{\rho_0} - \langle\lambda\rangle\frac{Q}{A_G} \qquad (2)$$

Making the difference (1)-(2):

We obtain:

$$\frac{\partial u}{\partial t} + \lambda_0 u = \frac{1}{A_G}\frac{\partial Q}{\partial t} + \lambda_0\frac{Q}{A_G} + \left(\frac{g\Delta\rho}{L\rho_0} + \frac{\alpha\lambda_0 Q}{H_G A_G}\right)\left(z + \frac{H_G}{2}\right).$$

The Velocity equation (Harzallah,2009)

Transformed to the spectral domain using *u, Q, Δρ* ~ $e^{i\omega t}$

$$\tilde{u} = \frac{\tilde{Q}}{A_G} + \boxed{\frac{\lambda_0^2 - i\lambda_0\omega}{\lambda_0^2 + \omega^2}} \left(\frac{g\Delta\tilde{\rho}}{L\lambda_0\rho_0} + \frac{\alpha\tilde{Q}}{H_G A_G} \right) \left(z + \frac{H_G}{2} \right)$$

The z-dependent term $\lambda_0^2 - i\lambda_0\omega / \lambda_0^2 + \omega^2$

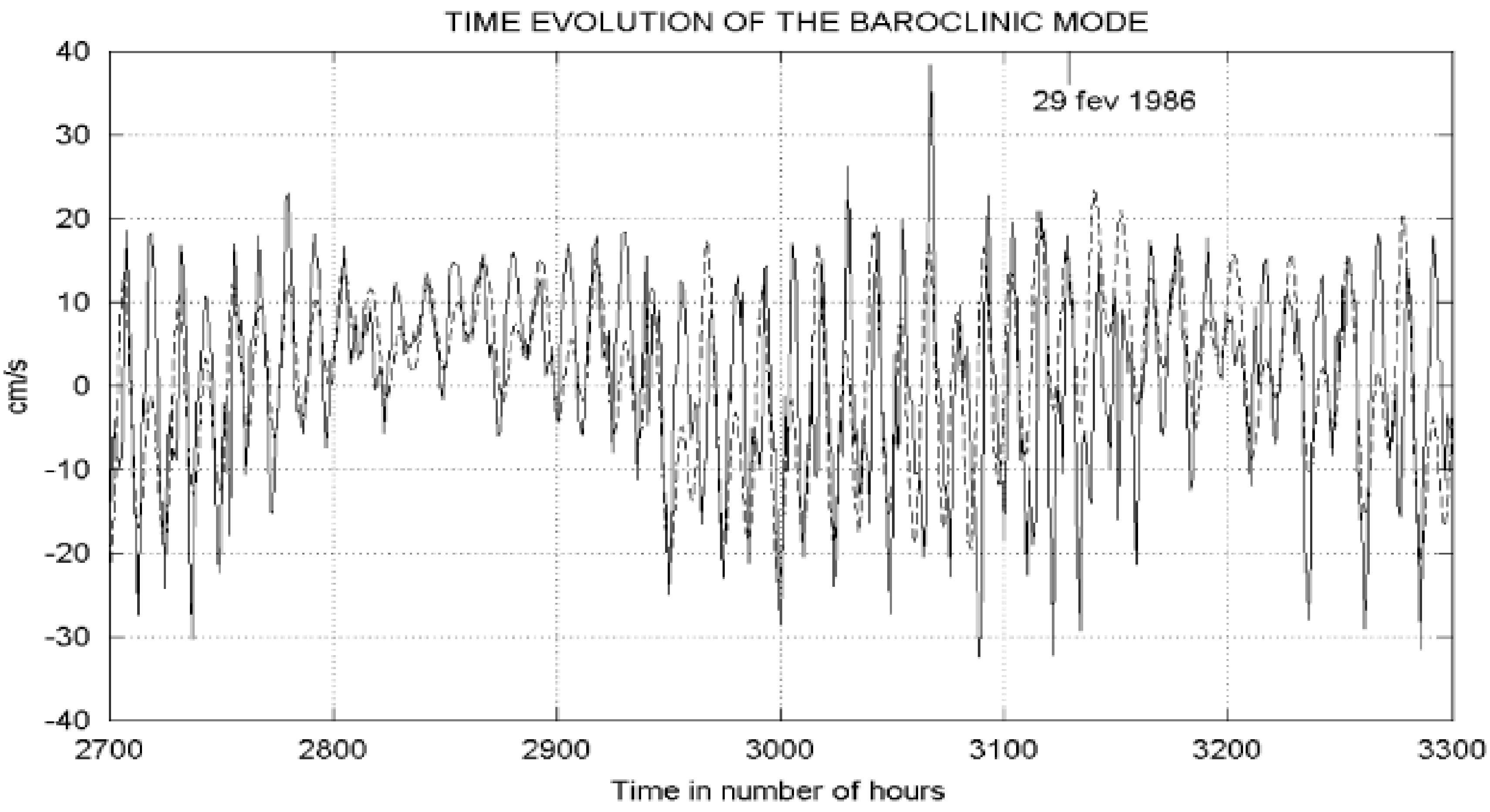
ω → 0 : amplitude → 1 and phase → 0

ω ↗ : amplitude → 0 and phase → +π/2

Model verification (Harzallah,2009)

Used values: $\lambda_0 = 5.1 \times 10^{-5}\,\mathrm{s}^{-1}$, $\alpha = 1.2$ and $\langle \lambda \rangle = 8.2 \times 10^{-5}\,\mathrm{s}^{-1}$

Present analytical model is compared to observations (second eigenvector)



The interface equation (Harzallah,2009)

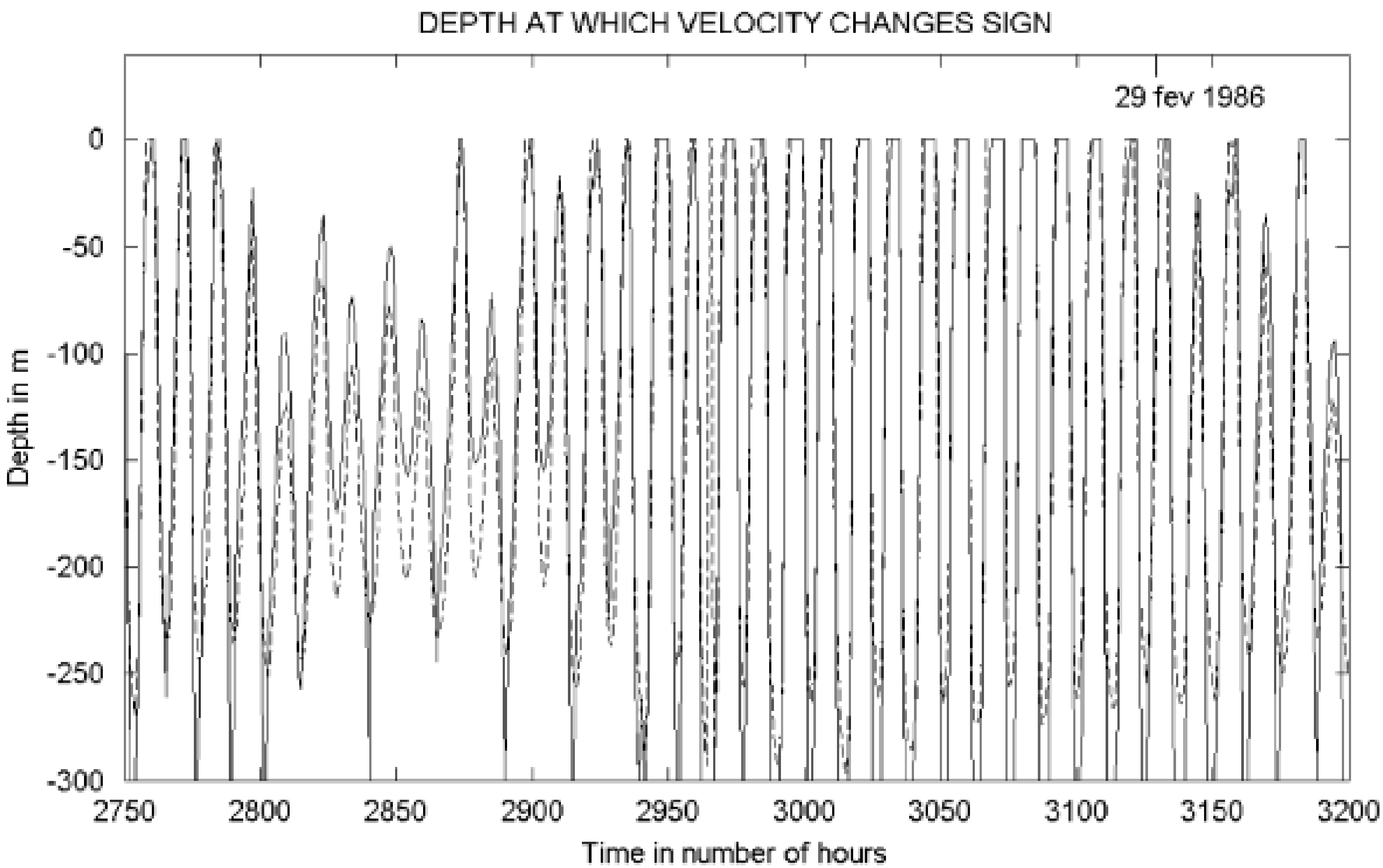
Interface depth : $z(u=0)$

$$z(u=0) = -\frac{H_G}{2} - \frac{\lambda_0 + i\omega}{\lambda_0} \frac{H_G}{(A_G H_G g \Delta\rho / L\lambda_0 Q \rho_0 + \alpha)}.$$

Not obvious, we consider M2+S2 (Ts)

$$\omega = 2\pi/T_s \text{ with } T_s \sim 12.42\ \text{h},$$

Validation (Harzallah,2009)



Dynamic and sterif effects (Harzallah, 2018, sub.)

Dynamical effects:

We consider again (2) without *Pa*:

$$\frac{1}{A_G}\frac{\partial Q}{\partial t} = -\frac{g\eta_M}{L} - \frac{gH}{2L}\frac{\Delta\rho_D}{\rho_0} - \langle\lambda\rangle\frac{Q}{A_G}.$$

$\Delta\rho_D$ is the density difference through the Strait due to Strait control

Using the conservation equation $\frac{\partial\eta_M}{\partial t} = \frac{Q}{A_M}$

we obtain

$$\frac{A_M}{A_G}\frac{\partial^2\eta_M}{\partial t^2} = -\frac{g\eta_M}{L} - \frac{gH_G}{2L}\frac{\Delta\rho_D}{\rho_0} - \langle\lambda\rangle\frac{A_M}{A_G}\frac{\partial\eta_M}{\partial t}$$

Dynamic and sterif effects (Harzallah, 2018, sub.)

Steric effects of the Atlantic :

$$\frac{1}{A_G}\frac{\partial Q}{\partial t} = -\frac{g(\eta_M - \eta_A)}{L} - \langle\lambda\rangle\frac{Q}{A_G} \qquad \frac{\partial \eta_M}{\partial t} = \frac{Q}{A_M} \qquad \frac{\partial \eta_A}{\partial t} = -\frac{H}{\rho_0}\frac{\partial \rho_A}{\partial t}$$

Leads to

$$\frac{A_M}{A_G}\frac{\partial^2 \eta_M}{\partial t^2} = -\frac{g(\eta_M - \eta_A)}{L} - \langle\lambda\rangle\frac{A_M}{A_G}\frac{\partial \eta_M}{\partial t}$$

ρ_A is the Atl. density

Steric effetcs of the Mediterranean:

variations with the corresponding conservation equation:

$$\frac{1}{A_G}\frac{\partial Q}{\partial t} = -\frac{g\eta_M}{L} - \langle\lambda\rangle\frac{Q}{A_G} \qquad \frac{\partial \eta_M}{\partial t} = \frac{Q}{A_M} - \frac{H}{\rho_0}\frac{\partial \rho_M}{\partial t}$$

ρ_M is the Med. density

We obtain

$$\frac{A_M}{A_G}\frac{\partial^2 \eta_M}{\partial t^2} = -\frac{g\eta_M}{L} - \langle\lambda\rangle\frac{A_M}{A_G}\frac{\partial \eta_M}{\partial t} - \frac{A_M}{A_G}\frac{H}{\rho_0}\frac{\partial^2 \rho_M}{\partial t^2} - \langle\lambda\rangle\frac{A_M}{A_G}\frac{H}{\rho_0}\frac{\partial \rho_M}{\partial t}$$

Dynamic and sterif effects (Harzallah, 2018, sub.)

We transform to the spectral domain

$$\eta_M = -F(\omega)\frac{H}{2}\frac{\Delta\rho_D}{\rho_0}$$

$$\eta_M = -F(\omega)\frac{H}{\rho_0}\rho_A$$

$$\eta_M = -\left(1 - F(\omega)\right)\frac{H}{\rho_0}\rho_M$$

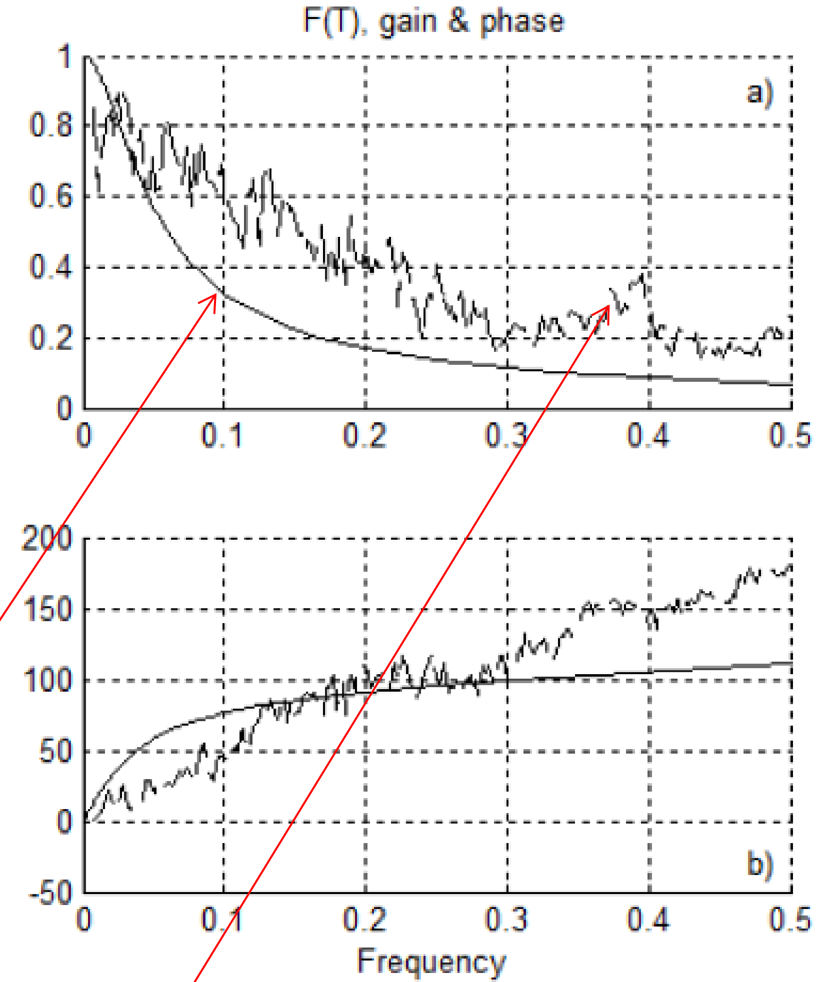
with $F(\omega) = \dfrac{1-\beta\omega^2 - i\langle\lambda\rangle\beta\omega}{(1-\beta\omega^2)^2 + (\langle\lambda\rangle\beta\omega)^2}$

and $\beta = \dfrac{L}{g}\dfrac{A_M}{A_G} = 5\ 10^{+9}.$

Dynamic and sterif effects (Harzallah, 2018, sub.)

$$\eta_M = -F(\omega)\frac{H}{2}\frac{\Delta\rho_D}{\rho_0}$$

$$F(\omega) = \frac{1-\beta\omega^2 - i\langle\lambda\rangle\beta\omega}{(1-\beta\omega^2)^2 + (\langle\lambda\rangle\beta\omega)^2}$$



amplitude (a) and phase (b) of F

gain (a) and phase (b) between $-\frac{H}{2}\frac{\Delta\rho_D}{\rho_0}$ and η_M

Based on the INSTMED06 3D complete thermodynamics Med. numerical model 1958-2007

Validation (Harzallah, 2018, sub.)

In $F(\omega) = \dfrac{1-\beta\omega^2 - i\langle\lambda\rangle\beta\omega}{(1-\beta\omega^2)^2 + (\langle\lambda\rangle\beta\omega)^2}$

$-\beta\omega^2$ << for T larger than few weeks

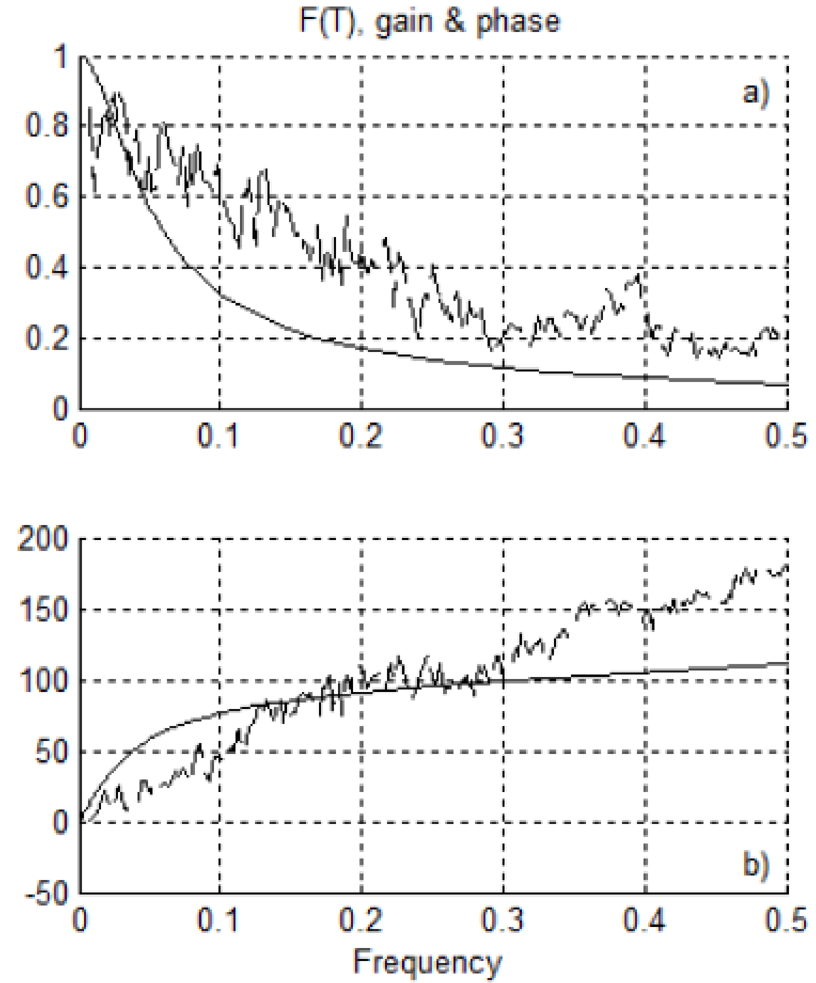
$\langle\lambda\rangle\beta\omega$ << for T larger than few months

$F(\omega) \rightarrow 1 \text{ when } \omega \rightarrow 0$

$$\eta_M = -F(\omega)\frac{H}{2}\frac{\Delta\rho_D}{\rho_0}$$

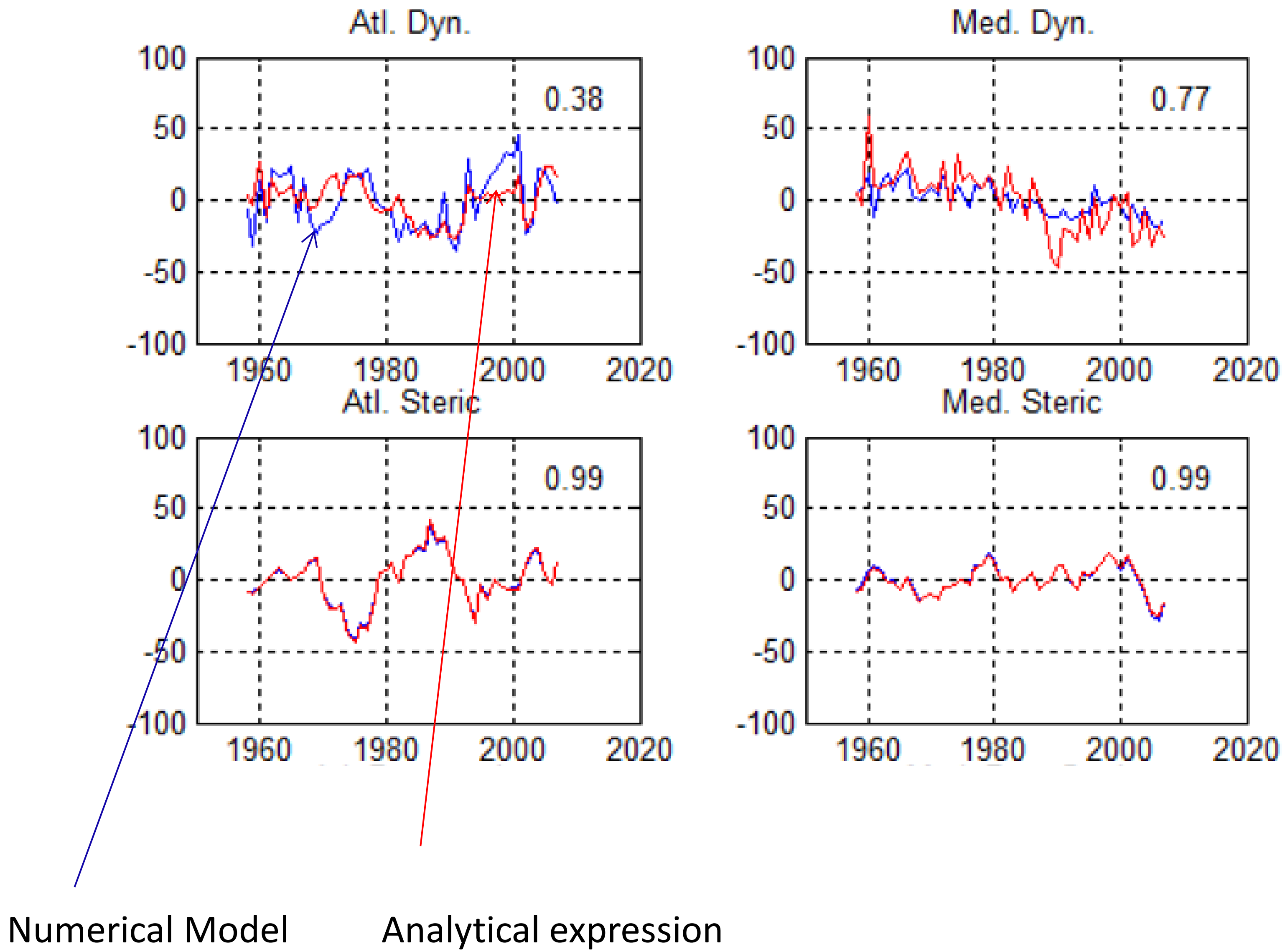
$$\eta_M = -F(\omega)\frac{H}{\rho_0}\rho_A$$

$$\eta_M = -(1-F(\omega))\,\frac{H}{\rho_0}\rho_M$$



$$\eta_M = -\frac{H}{2}\frac{\Delta\rho_D}{\rho_0} - \frac{H}{\rho_0}\rho_A$$

Validation (Harzallah, 2018, sub.)



CONCLUSION

The Mediterranean basin is a semi-enclosed basin with one connection to the Atl.

Complex dynamical behaviour at this connection

An analytical model has been developed and validated against a numerical one

A unique function $F(\omega)$ emerges ; it provides a practical description of the different behaviours in the Med.