Workshop on PDE's Modelling & Theory

09-10 May 2018

Palais des Sciences de Monastir

Tunisie

Program and abstracts

UR Analysis and Control of PDEs, University of Monastir, Tunisia

Organising comittee

Kaïs AMMARI, UR Analysis and Control of PDE, University of Monastir, Tunisia
Faouzi TRIKI, Laboratoire Jean Kuntzmann, Université de Gronoble Alpes, France

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Workshop on PDE's: Modelling and Theory

Wednesday 09 May 2018

9h-9h15 : Welcoming participants

9h15-9h20 : Opening of the workshop

9h20-9h55 : E. Bonnetier (Grenoble)

9h55-10h30 : A. Bchatnia (Tunis)

10h30-10h50 : Coffee Break

10h50-11-25 : D. Volkov (Worcester)

11h25-12h : A. Boughammoura (Monastir)

12h-14h : Lunch

14h30-15h05 : F. Guevara Vasquez (Utah)

15h05-15h40 : R. Assel (Monastir)

15h40-16h : Coffee Break

16h-16h35 : A. Duca (Grenoble)

16h35-17h10 : C. Castro (Madrid)

17h10-17h45 : A. Boumenir (Georgia)

17h45-18h20 : Poster Session

18h20-20h : Guided tour of the Sciences Palace and the Medina

20h: Dinner Gala

Thursday 10 May 2018

9h-9h35 : C. Amrouche (Pau)

9h35-10h10 : M. Hassine (Monastir)

10h10-10h45 : A. Harzallah (Monastir)

10h45-11h : Coffee Break

11h-11h35 : M. Choulli (Metz)

11h35-12h10 : T. Yao (Grenoble)

12h10-12h45 : M. Ismail (Grenoble)

12h45: Closing and lunch

Talks

Chérif AMROUCHE
Université de Pau, France

« Stokes and Navier-Stokes equations with Navier boundary condition »

Mots-clés: Stokes equations, Navier boundary condition, weak solution, L^p -regularity, limiting case

We focus on the study of the incompressible fluid in a bounded domain in $\mathbb{R}^3$. We consider the stationary Stokes equation

$$-\Delta \square + \nabla\pi = f, \ \text{div} \ \square = 0 \text{ in } \Omega \qquad (1)$$

and the stationary Navier-Stokes equation

$$-\Delta \square + (\square \ . \ \nabla)u + \nabla\pi = f, \ \text{div} \ \square = 0 \text{ in } \Omega \qquad (2)$$

where Ω be a bounded domain in $\mathbb{R}^3$ with boundary Γ, possibly not connected, of class $C^{1,1}$ and $\square$ and π are the velocity field and the pressure of the fluid respectively, f is the external force acting on the fluid. A boundary condition was suggested by Navier (in 1823) which states that on one hand, the normal component of the fluid velocity is zero at the boundary (impermeability condition) and on the other hand, the amount of slip in the tangential part of the velocity, rather than being zero, is proportional to the tangential part of the normal stress exerted by the fluid on the boundary i.e.

$$\square \ . \ n = 0, \ \ 2\,[(\mathbb{D}\square)n]_\tau \ + \alpha \ \ \square_\tau = 0 \text{ on } \Gamma \qquad (3)$$

where n and τ are the unit outward normal and tangent vectors on Γ respectively and $\mathbb{D}\square = \frac{1}{2}(\nabla+\nabla^T)$ is the rate of strain tensor. Here, α is the coefficient which measures the tendency of the fluid to slip on the boundary, called friction coefficient. We are interested to discuss the well-posedness of the problem (1) and (2) along with (3), in particular existence, uniqueness of weak and strong solutions in $\mathbf{W}^{1;p}(\Omega)$ and $\mathbf{W}^{2;p}(\Omega)$ for all $1 < p < \infty$ considering minimal regularity on the friction coefficient α. Moreover, we deduce estimates to analyze the behavior of the solution with respect to α which indicates in some sense, an inverse of the derivation of the Navier boundary conditions from no-slip boundary condition for rough boundaries.

Rachid ASSEL
Université de Monastir, Tunisie

« Control of some infinite networks »

We present some recent results on the energy behavior for the damped wave equation in some infinite networks. We use spectral theory methods to study the decay of the energy. The exponential decay rate is computed when the network has an equilateral finite part.

Ahmed BCHATNIA
Université Tunis El Manar, Tunisie

« Observability and stabilization of the wave equation with moving boundary »

We deal with the wave equation with assigned moving boundary $(0 < x < a(t))$ upon which Dirichlet or mixed boundary conditions are specified. Here $a(t)$ is assumed to move slower than light and periodically.
First, we prove an observabilty estimates based on the transverse velocity at a(t) and we deduce the exact controllability for an example of the function $a(t)$. Second, we give a feedback which guarantees the exponential decay of the energy. Finally, we give a remark on the moving-pointwise stabilization problem.

Eric BONNETIER
Université de Grenoble Alpes, France

« Plasmonic resonances in planar domains with corners' »

The quasistatic resonances of a metallic particle are defined as the eigenelements of the Neumann-Poincar\'e operator, an integral operator that proves quite useful in the study of 2nd order PDE's with piecewise constant coefficients.
In dimension 2, when the particle has the shape of a domain with corners, this operator has essential spectrum. We relate this essential spectrum to the theory of elliptic corner singularities. In particular, this provides insight on the behavior of the associated generalized eigenfunctions, and how they localize around the corners.
This is joint work with Hai Zhang (Hong Kong University of Science and Technology).

Ahmed BOUGAMMOURA
Université de Monastir, Tunisie

« Homogenization of an elastic medium having three phases »

We study an elastostatic problem in an ε-periodic medium having threephases: matrix, fibers, and fiber coatings. The rigidity is of order one along the fibers and is scaled by ε^2(the so-called double porosity scaling) in both the transverse directions and the fiber coatings. Using the homogenization process, we show that both the effective transverse traction and the longitudinal stress in the fibers are mainly influenced by the elastic properties of the fiber coatings.

Amin BOUMENIR
University of West Georgia, USA

« One point recovery of a parabolic equation »

We recover the shape of the domain and the diffusion coefficient of a parabolic equation from observations of an infinite sequence of solutions taken at an arbitrary single point inside or on the boundary of the unknown domain. The method is to reconstruct the principal eigenfunction of the associated elliptic operator from the observed data.

Carlos CASTRO
Politechnica de Madrid, Spain

« On the numerical approximation of the best decay rate
On the numerical approximation of the best decay rate for some dissipative systems »

It is well known that, under certain hypotheses, the spectrum of a dissipative system characterizes the best decay rate of the solutions.
We propose a numerical method to approximate the spectrum of some dissipative systems that are bounded perturbations of unbounded skew-adjoint operators. Roughly speaking, it consists in a projection method on finite dimensional subspaces generated by the first eigenfunctions of the unperturbed operator. As far as we know, the convergence of this projection method in this context has been considered for the first time by J. Osborn to approximate a single eigenvalue (or a fixed number of them) when the unperturbed operator is selfadjoint. In the present situation, we deal with perturbations of skew-adjoint operators but the result is easily generalized. We show that, under certain hypotheses, the analysis can be adapted to give a uniform approximation of the whole spectra (and not only a fixed number of eigenvalues) and therefore it can be used to approximate the spectral abscissa of the continuous model from the discrete one. We illustrate the numerical approach with several experiments for damped wave and beam models. This is a joint work with K. Ammari from University of Monastir (Tunez).

Mourad CHOULLI
Université de Lorraine-Metz, France

« Inverse elliptic and parabolic boundary value problems with partial data »

I will consider the problem of determining unknown coefficients in elliptic BVP and parabolic IBVP from the knowledge of a partial Dirichlet-to-Neumann map. I will present some recent stability estimates based on joint works with Yavar Kian and Eric Soccorsi. These stability estimates rely on the construction of complex geometric optic solutions vanishing at a part of the boundary.

Ali HARZALLA
INSTM-Monastir, Tunisie

« The Mediterranean Sea level variability from an ensemble of numerical model simulations and analytical expressions »

The Mediterranean Sea level is split into dynamic and expansion/contraction (steric) parts and to effects originating from the Mediterranean basin and the Atlantic Ocean. A numerical model is used to simulate the Mediterranean Sea level under different configurations. The results show that the model can be used to demonstrate types and origins of the sea level variability in the Mediterranean Sea. It is shown that periods ranging from few to tens of years dominate the variability and are mainly related to hydrographic conditions in the Atlantic area. Dynamical effects related to the dynamic effects of the Mediterranean hydrographic conditions induce a rather continuously decreasing sea level. The simulations also show steric sea level variability with opposite sign fluctuations to those related to the flow at the Strait of Gibraltar. Analytical models are also derived to reproduce the contributing terms to the sea level in the Mediterranean Sea based on expressions of Harzallah (2009). The analytical model expressions are transformed to the spectral domain and the results show realistic resemblance to observations. The approach followed here constitutes an interesting approach for the investigation of the sea level variability in semi-enclosed basins as the Mediterranean Sea.

Maatoug HASSINE
Université de Monastir, Tunisie

« Topological sensitivity analysis based method for solving some geometric inverse problems »

The topological sensitivity analysis method consists in studying the variation of a given shape function with respect to the creation of a small geometric perturbation. In this talk, we present a topological asymptotic expansion for a parabolic operator valid for a large class of shape functions and an arbitrary shaped geometric perturbation. The leading term of the shape function variation is used for constructing a numerical optimization algorithm. The efficiency and the accuracy of the obtained algorithm are illustrated by some numerical applications.

Alessandro DUCA
Université de Grenoble Alpes, France

« Global exact controllability of the bilinear Schrödinger equation on compact graphs »

In quantum mechanics any pure state of a system is mathematically represented by a wave function ψ in the unit sphere of a Hilbert space $\mathscr{H}$.

The dynamics of a particle constrained in a compact graph structure $\mathscr{G}$ and excited by a controlled field is represented by the Cauchy problem in $\mathscr{H}=L2(\mathscr{G}, \mathbb{C})$

$$(1) \qquad \begin{cases} i\partial_t\psi(t) = A\psi(t) + u(t)B\psi(t), \\ \psi(0) = \psi^0, \qquad t \in (0,T). \end{cases}$$

The operator B is bounded symmetric, u is a control function and ψ^0 is the initial state of the system. The operator $A = -\Delta$ is the Laplacian equipped with self-adjoint type boundary conditions into the vertices of the graph.
We study the controllability of the bilinear Schrödinger equation on compact graphs and we analyze how the boundary conditions and the structure of the graph affect the result.
Provided the well-posedness of (1), we present assumptions on B and on the spectrum of A implying the global exact controllability in suitable subspaces of $\mathscr{H}$.
When the previous assumptions fail, we introduce the so-called "energetic controllability", which allows to provide controllability results also when $\mathscr{G}$ is a complex structure and the global exact controllability is not verified.

Fernando GUEVARA VASQUEZ
University of Utah, USA

« Imaging with Stokes parameters »

We present a method for imaging small scatterers within a homogeneous medium from measurements of the Stokes parameters (polarization state).
The measurements are obtained by illuminating the medium with waves emanating from a dipole source of known position and polarization. We show that the polarization data can be used to estimate the electric field at the array. We use a stationary phase argument to show that the estimated electric field can be used instead of the electric field to image the scatterers with an electromagnetic version of

Kirchhoff migration, yielding images that are identical in a high frequency limit. The Kirchhoff images can be used to estimate a projection of the polarizability tensor of the scatterers in an appropriate basis.

Mourad ISMAIL
Université de Grenoble Alpes, France

« Quelques modèles de fluides complexes. Applications aux écoulements sanguins »

Je présenterai quelques modèles de fluides complexes et leurs applications aux écoulements sanguins.
Dans ce cadre, le sang est considéré comme un fluide newtonien simple (le plasma) dans lequel baigne une suspension de globules rouges. Ces derniers sont assujettis à des force élastiques et de courbure et modélisés grâce à la méthode Level Set. Ainsi, on considère un couplage des équations de Navier-Stokes avec une équation de transport (pour les globules rouges).
Je présenterai également l'implémentation de ces modèles dans un cadre de calcul haute performance en utilisant notre librairie libre Feel++ http://www.feelpp.org basée sur la méthode des éléments finis.

Darko VOLKOV
Worcester State University, USA

« Reconstruction of faults in elastic half space from surface measurements »

We study a half space linear elasticity model for surface displacements caused by slip along underground faults. We prove uniqueness for the inverse problem aimed at recovering the fault location and (piecewise planar) geometry and of the slip field for a given surface displacement field. We then discuss recently derived Lipschitz stability results for the geometry of the fault. Next, we introduce a reconstruction algorithm for the realistic case where only a finite number of surface measurements are available. We prove that the minimum of a related error functional converges to the unique solution of the related fault inverse problem. Due to inherent uncertainties in measurements, rather than seeking a deterministic solution to the fault inverse problem, we consider a Bayesian approach. In this approach the geometry of the fault is assumed to be planar, it can thus be modeled by a three dimensional random variable whose probability density has to be determined knowing surface measurements. We prove convergence results on the probability density obtained from the Bayesian model for small standard deviation of the data.
This Bayesian approach leads to a very large computation since the slip is unknown. To contend with the size of this computation we developed an algorithm for the numerical solution to the stochastic problem which can be easily implemented on a parallel multi-core platform and we discuss techniques aimed at saving on computational time. After showing how this algorithm performs on simulated data, we apply it to measured data. The data was recorded during a slow slip event in Guerrero, Mexico.

Tao YIN
Université de Grenoble Alpes, France

« Inverse source problems in elastodynamics »

We are concerned with time-dependent inverse source problems in elastodynamics. The source term is supposed to be the product of a spatial function and a temporal function with compact support. We present frequency-domain and time-domain approaches to show uniqueness in determining the spatial function from wave elds on a large sphere over a finite time interval. Stability estimate of the temporal function from the data of one receiver and uniqueness result using partial boundary data are proved. Our

arguments rely heavily on the use of the Fourier transform, which motivates inversion schemes that can be easily implemented. A Landweber iterative algorithm for recovering the spatial function and a non-iterative inversion scheme based on the uniqueness proof for recovering the temporal function are proposed.This is a joint work with G. Bao, G. Hu and Y. Kian.

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